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10th International Congress on
Industrial and Applied Mathematics August 20-25, 2023

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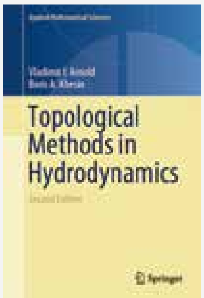

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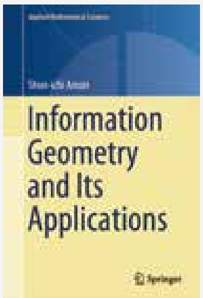

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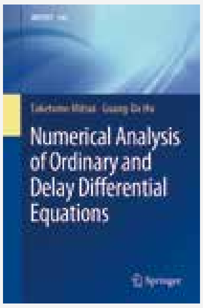

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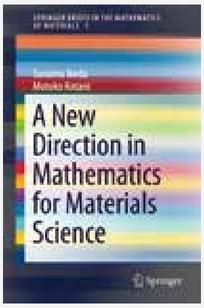

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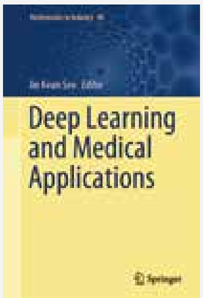

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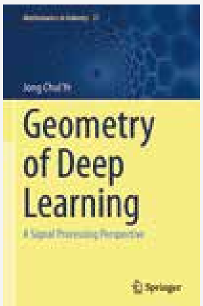

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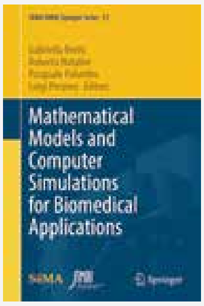

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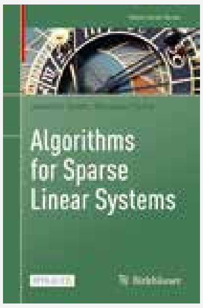

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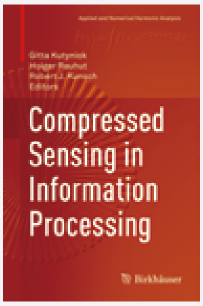

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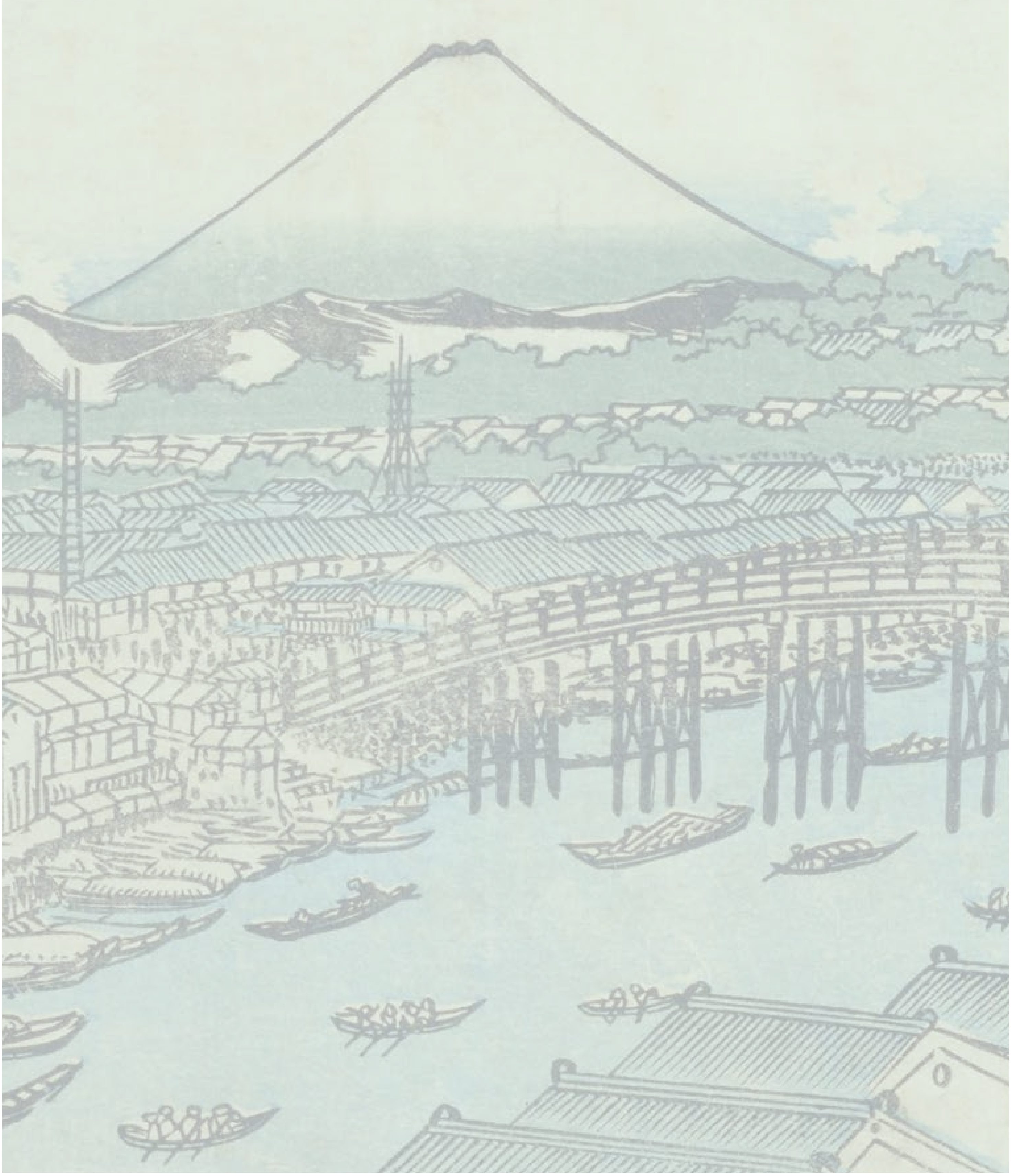


Ya-xiang Yuan

ICIAM President

Welcome Message to ICIAM 2023

On behalf of the International Council for Industrial and Applied Mathematics, I would like to welcome all of you to ICIAM 2023, and to Tokyo, the beautiful capital city of Japan. I hope you will have a wonderful time attending interesting talks, engaging in stimulating discussions, meeting old friends and making new ones.

ICIAM congresses are known for their rich programme , and I am proud to say that this one is no exception. As we ring up the curtain on the congress, please allow me to highlight a few items on the agenda this year.

At the opening ceremony, we shall present to you the winners of the 2023 ICIAM prizes. Although the winners were already announced on September 19th, 2022, it is only at the opening ceremony of the congress that they are officially awarded. This is an opportunity to acknowledge the achievements and appreciate the contribution of these outstanding colleagues. The prize winners will present their research work during the course of the congress.

Besides the talks by this year's ICIAM prize winners, there will be 27 invited talks, selected by the Scientific Program Committee, chaired by Professor Yasumasa Nishiura. The Scientific Program Committee has done a wonderful job in selecting these 27 applied mathematicians' work in different areas, showcasing the diversity of the topics represented at ICIAM congresses.

Another highlight of the week will be the Olga Taussky-Todd lecture, also known as the OTT lecture, delivered this time by Professor Ilse C.F. Ipsen of North Carolina State University from the United States. The OTT lectures, given for the first time in 2007 in Zurich, were set up by ICIAM upon the suggestions of the AWM and EWM societies with the intent to celebrate a female scholar who has made outstanding contributions to applied mathematics and/or scientific computation. The lectures pay tribute to Olga Taussky-Todd, a great female mathematician whose scientific legacy belongs in both theoretical and applied mathematics and whose work exemplifies the qualities we want to recognize in this lecture series. The OTT lecture has long been sustained by small individual donations, and this year we deeply appreciate the generous donations from Mathworks, Comsol and the Inamori Foundation from Japan.

There will be two public lectures at ICIAM 2023. One is given by Professor Noboru Kikuchi, the Genesis Research Institute Inc. The other public lecture will be given by Professor Padmanabhan Seshaiyer of George Mason University from the United States, under the title "Understanding the Dimensions of Justice, Equity, Diversity and Inclusion (JEDI) Across the Globe in Applied Mathematics Research and Education". Professor Padmanabhan Seshaiyer works in the broad areas of computational and applied mathematics, computational data science, computational biomechanics and STEM Education. During the last two decades, he has initiated and directed a variety of educational programs including faculty development; post-graduate, graduate and undergraduate research; K-12 outreach; teacher professional development; and enrichment programs to foster the interest of students and teachers at all levels to apply well-developed research concepts to fundamental applications arising in STEM disciplines. I have no doubt these two talks will be especially fascinating to mathematicians and to the general public alike.

Pioneered by the Valencia Congress four years ago, an "Industry Day" will be held on August 23rd. The Industry Prize lecture and industry-related invited lectures will be presented on that day, in addition to many other events centered around industrial applications and collaborations. We are thankful to the many industry professionals who have helped with the organization of "Industry Day" and to the congress in general. I have no doubt that "Industry Day" will be one of the main attractions of ICIAM congresses that many of us look forward to.

Apart from the invited talks and distinguished lectures, there will be many mini-symposia, contributed talks and poster sessions, covering all areas of industrial and applied mathematics. In particular, JSIAM (The Japan Society for Industrial and Applied Mathematics), CSIAM (China Society for Industrial and Applied Mathematics) and SIAM (Society for Industrial and Applied Mathematics) will host their respective annual meetings, with events embedded into the agenda of the ICIAM Congress. These events are sure to complement each other on various levels.

Last but not least, my deepest gratitude goes to the organizing committee of ICIAM 2023, JSIAM, and the entire Japanese applied mathematics community who directly or indirectly supported the success of this congress. I would like to thank them for their tremendous efforts in assembling and coordinating the programme , and their kind hospitality in these wonderful conference facilities in Waseda University. It is remarkable how the organizing team brought together mathematicians from all over Japan and spearheaded the effort to host satellite events in a number of other Japanese cities outside of Tokyo, including Kyoto.

We look forward to having a successful congress, and I hope all the attendees will enjoy and benefit from ICIAM 2023 in Tokyo.

Shin'ichi Oishi

Congress Director, ICIAM 2023

Welcome to ICIAM 2023

ICIAM 2023 is organized by the Japan Society for Industrial and Applied Mathematics (JSIAM), the Mathematical Society of Japan (MSJ), and the Science Council of Japan (SCJ).

In April 1990, JSIAM was established to support and collaborate activities related to applied mathematical research, industry, and education in Japan. JSIAM oversees the editing of the Handbook of Applied Mathematics, Asakura Publishing Co., Ltd.

The MSJ was founded in 1877 as Tokyo-Sugaku-Kaisha (Tokyo Mathematics Society). It was expanded to Tokyo-Sugaku-Buturi-Kaisha (Tokyo Mathematical and Physical Society), and later to the Physico-Mathematical Society of Japan. In 1946, the society split into The Mathematical Society of Japan and The Physical Society of Japan. The MSJ oversees the editing of the Encyclopedic Dictionary of Mathematics, published by Iwanami Shoten.

In January 1949, the SCJ was established as the representative organization of the Japanese scientific community, subsuming the humanities, social sciences, life sciences, natural sciences, and engineering.

On behalf of the Japanese scientific community, the organizers of ICIAM 2023 warmly welcome participants from over 87 countries to Tokyo, Japan, where the event will be held from August 20th to 25th. Until last September, due to Covid-19, it was not possible to predict the conference format for ICIAM 2023. Fortunately, ICIAM 2023 will be held in person. In addition, all sessions will be available for online participation. The organizers of ICIAM 2023 eagerly anticipate the engagement of participants in fruitful exchanges of research ideas.

ICIAM 2023 will feature 27 invited talks, one Olga Taussky-Todd plenary lecture, six plenary talks from ICIAM prize recipients, and two plenary public lectures. These, together with 477 mini-symposia, will encompass almost 4,000 talks, more than 1,100 contributed talks, and approximately 400 poster presentations. Approximately 20 exhibitions will be held.

Thanks to the Tokyo Convention and Visitors Bureau (TCVB), we are able to provide financial support for registration fees, accommodation, and travel expenses for approximately 500 attendees from around the world. The organizers would like to extend their sincere thanks to the presenters of talks, as well as to the ICIAM officers led by President Yuan Ya-xiang, the members of the scientific program committee chaired by Professor Yasumasa Nishiura, and the members of the local programming committee headed by Professor Takeshi Ogita. Additionally, they express their gratitude to SIAM for managing the SIAM prize lectures, and organizing several mini-symposia based on SIAM activities.

ICIAM 2023's venue at Waseda University has been provided free of charge. This has enabled us to set the registration fee at a reasonable price. In 1882, just five years after the foundation of the MSJ, Waseda University was founded by Okuma Shigenobu. Okuma served two terms as the Prime Minister of Japan, and promoted numerous scientific research initiatives and events. Notably, he gathered support for Shirase Nobu to explore the Antarctic. Shirase started his exploration in 1910, but was unable to reach the Geographic South Pole. One year before, in 1909, the Faculty of Science and Engineering at Waseda University was established at Waseda's main campus. This was achieved through Okuma's philosophy. He believed that scientific and engineering innovations are necessary for maintaining a healthy society. However, establishing and managing a Faculty of Science and Engineering at a private university posed a challenge in Japan 125 years ago. Fortunately, due to strong support from Takeuchi Meitaro, the Faculty of Science and Engineering was able to take off. Incidentally, Takeuchi was the founder Komatsu Ltd., and the first commercially available automobile in Japan bears part of his name. This car was completed in 1914, and was named the DAT or Datsun. The letter 'T' comes from Takeuchi. In 1934, the company became Nissan. In 1949, the same year that the SCJ was founded, the Department of Mathematics was established within the Faculty of Science and Engineering. In 1967, the Faculty of Science and Engineering relocated to the Nishi-Waseda Campus in the Shinjuku ward of Tokyo, which is located approximately 15 minutes, by foot, from the main campus. In 2007, the Department of Applied Mathematics was established within the Faculty of Science and Engineering. The current President of Toyota, Koji Sato, graduated from the Faculty of Science and Engineering at Waseda University. The current Prime Minister of Japan, Fumio Kishida, is also a graduate of Waseda University.

The organizers would like to once again extend their gratitude to TCVB for their support in various aspects related to ICIAM 2023, including their generous provision of complimentary Tokyo sightseeing tours.

The organizers would like to thank everyone who supported ICIAM 2023, with special recognition to the Secretariat Group led by Professor Naoya Yamanaka, for their contribution in making ICIAM 2023 a vibrant platform for researchers in industrial and applied mathematics.

Hiroshi Suito

Tohoku University, Japan

Student Research Programs in Collaboration with Industry

This article introduces a student research program with industrial projects in mathematics, called "g-RIPS-Sendai", which has been held in Japan since 2018. The Research in Industrial Projects for Students (RIPS) program is a long-time activity held at the Institute for Pure and Applied Mathematics (IPAM), at the University of California, Los Angeles (UCLA) `http://www.ipam.ucla.edu/programs/student-research-programs/`.

In 2017, after holding discussions with people at IPAM, we decided to start a similar program in Japan. After establishing it as a "graduate-level" program (g-RIPS) considering school calendars in Japan, we have been continuing and extending this program in collaboration with IPAM and Japanese industrial enterprises.

This program offers graduate students in mathematics and related areas stimulating opportunities to work on realistic research projects in industries. Students from Japan and the U.S. work in cross-cultural teams for eight weeks in the summer on research problems provided by industrial partners. Workplaces for the projects are allocated at the Katahira campus of Tohoku University. Hotel accommodations are provided in Sendai city. Projects involving both analytic and computational aspects are of intense interest to industrial partners. They are not merely easy challenges for graduate students. Each team usually includes two students from the U.S. and two from Japan, who are supported continuously by industrial mentor(s) from the industrial partner and academic mentor(s) from Tohoku University. English is the only language used during the program. Students come from different backgrounds and have different expertise and usually complement each other. After an initial learning phase, the students define the goals and milestones of the project. They work diligently with various kinds of support from mentors toward presenting their progress at the mid-term and final presentations with complete final reports. During the program, students have other opportunities to visit industrial partners' firms or institutions and to participate in Japanese language and culture courses. After starting this program in 2018, we were compelled to cancel it in 2020 because of the COVID-19 pandemic. Nevertheless, we held the program in online-style, managing the time difference between the U.S. and Japan in 2021, and subsequently resuming the in-person style in 2022.

We are grateful to our industrial partners, who have been providing research topics, necessary datasets, funding for student accommodations, and other forms of support. Our industrial partners, Fujitsu Co. Ltd., IHI Corp., Mitsubishi Electric Corp., and NEC Corp. are all major Japanese companies. F-MIRAI is a research center established at Tsukuba University by Toyota Motor Corporation. A list of the project titles so far in the program is presented below with corresponding partners.

FY2018:
- F-MIRAI: Design for the next-generation energy and mobility platform.
- NEC: Reliable wireless networking systems for the industrial internet-of-things (IoT).

FY2019:
- F-MIRAI-A: Design for the next-generation mobility service in suburban areas – Mobility service for a university campus.
- F-MIRAI-B: Design for the next-generation mobility service in suburban areas – Mobility service for hospital guests.
- FUJITSU: Resolving real-world issues by "Digital Annealer".
- NEC: Combinatorial optimization using quantum annealing.

FY2021:
- F-MIRAI: Design for next-generation mobility service in suburban areas.
- MITSUBISHI ELECTRIC, Advanced Technology R&D Center: Development of a mapping space for intuitive teleoperation with heterogeneous devices of multiple types.
- MITSUBISHI ELECTRIC, Information Technology R&D Center: Optimization of wireless base station placement as an essential foundation for our future IoT society.
- NEC: Annealing machine application to artificial neural networks.

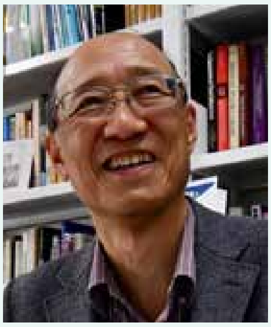
FY 2022:
- F-MIRAI: Mathematical approaches for mobility services in suburban areas.
- MITSUBISHI ELECTRIC, Advanced Technology R&D Center: Construction for incomplete map matching based on local and global geometries.
- MITSUBISHI ELECTRIC, Information Technology R&D Center: Multi-objective optimization for best early prediction of extreme weather events.
- NEC: Application of annealing machines to production planning optimization.

FY 2023:
- FUJITSU: Enhancing the explainability of modern AI.
- IHI: Mathematics for trajectory extrapolation using vehicle and human traffic data toward zero traffic fatalities.
- MITSUBISHI ELECTRIC, Advanced Technology R&D Center: Construction of metrics for map matching between travel trajectories and map graphs.
- MITSUBISHI ELECTRIC, Information Technology R&D Center: Novel technique to estimate wave spectra using ocean HF radar for environmental monitoring.
- NEC, Data Science Research Laboratories: Automated negotiation for supply chain management.

This program offers several different values for both students and industrial partners. For students, the program provides opportunities to be exposed to real-world problems and to grasp ideas about how mathematicians work in diverse industries. Students also gain valuable experience through collaboration with other students from a broad scope of specialties. The international and intercultural environment enhances their experiences with diversity. For industrial partners, the program provides chances to challenge themselves with new mathematical approaches and fresh ideas from young students, both for their own specific problems and for problems that are commonly confronted in the same business fields.

We have observed several different outcomes from this program. Sometimes projects have continued between the industrial partner and the team members after the program period has ended. Those and other efforts have been eventually published in scientific journals or introduced as presentations at international conferences. At other times, projects have been transformed into collaborative research between the company and the university researchers. Moreover, some students have gained employment in participating companies. These various outcomes are exciting characteristics of the program.

Participants in this program gather from many universities in both countries. Tohoku University will strengthen such activities and make appropriate contributions to enhancing collaborative activities between mathematics and industries.



Ichiro Tsuda

Chubu University, Japan

Interdisciplinary Elements Between Mathematics and Other Scientific Disciplines

Introduction

I come from a physics background, particularly, nonlinear and nonequilibrium statistical physics. Around 1977, I began exploring chaotic phenomena and then started to learn ergodic theorems, diffeomorphisms, and several types of stabilities, such as structural stability, attractors, strange attractors, and chaos within a framework of nonlinear dynamical systems (both topological and measure-theoretic), in the mathematics literature. I enthusiastically researched phenomenological chaos with help from the related mathematics. Fortunately, I identified several new mathematical phenomena such as noise-induced order[1] and chaotic itinerancy.[2, 3] Although I was progressing in chaos research, I decided to apply the mathematical theories of chaos in understanding the higher functions of the brain.

The standpoint that I took in brain science studies was to make a mathematical model of neural dynamics in terms of chaotic dynamical systems and then extract mathematical structures from experimental or numerical data. The purpose of this study was to construct an adequate language for understanding the brain, following Gelfand.[4] Three typical elements of interdisciplinary study in applied mathematics could be identified. This article on the elements in applied mathematics is designed to encourage young researchers to identify their own research methods.

Some Mathematical Elements

What is the difference between mathematics and other scientific disciplines? First, we must describe other scientific disciplines to be able to clearly elucidate the characteristics of mathematics. Some examples of research areas in physics include the treatment of the motion of objects and their universal laws. Modern physics began with Galileo's experiments for the motion of an object, including his later thought experiment, which were further developed via Newton's mathematical formulation of the motion of objects. Physics has since been established as the most rigorous natural science via mathematical formulation, which enables us to use experimental facts as a predictive set. This development was followed by the discoveries of electromagnetism, thermodynamics and statistical mechanics, hydrodynamics, relativity theory, quantum mechanics, and field theory. In addition, geophysics has developed as a complex physical system,[5] to clarify the multiple relationships between the factors that constitute the global environment. Chemistry has treated the molecular mechanisms of the change of materials via molecular interactions at the level of a complex of molecules. However, chemistry is not described in mathematical terms, except in a few research areas such as quantum chemistry. The main purpose of chemistry is the synthesis of materials. Similarly, biology is concerned with the molecular mechanisms of life, including the origin of life and also the relationship between structure and function in biological organisms. In addition, biology has clarified the evolution in the behaviors of living organisms leading to human behaviors. All these disciplines belong to natural science.

However, the social and human sciences have investigated individual behaviors and mental states, whereby the structure and dynamics of the society itself could become an object of study. Although human behaviors can be quantified by mathematics, such as economics, most disciplines do not use mathematics but instead use natural languages in each specific formalization.

Finally, engineering applies mathematics, physics, and chemistry to develop industrial technologies. One of engineering's central concepts is optimization, whereby the systems are controlled to achieve given goals.

Mathematics is based on the different elements from these disciplines. The term "mathematics" stems from Greek, μάθημα (or its plural form μάθηματα), which means what is learned, or knowledge itself. Thus, mathematics should include all human thought and behavior. In ancient days, the need for people (or their kings) to measure their occupied territory could be realized by evaluating small computable areas, such as the areas of a right triangle, rectangle, or square, by drawing parallel lines using the Pythagorean theorem, which later led to integral calculus in the calculation of areas, and also to geometry in the division of areas. Furthermore, counting numbers is based on the mental process of identification of different objects, by viewing them as the same class, that is, categorization, which may have led to algebra.

Another mathematical element is logical thinking. This element dates to the ancient Greek mathematician Euclid and his influential Greek text ΣτοιΧεῖα, which was translated into English as *Elements*. In my personal opinion, *Elements* provided proofs for a method that correctly operationalizes mental processes based on postulates or axioms. Boole and Turing later demonstrated the relationship between mathematics and thoughts. Boole's 1854 book, *An Investigation of the Laws of Thought on Which Are Founded the Mathematical Theories of Logic and Probabilities*,[6] reconstructed probability theory in terms of only two numbers, 0 and 1, which can express a truth value of logical thinking. Turing then investigated thoughts by simplifying their processes to "calculations on paper" and published an article entitled "On computable numbers, with an application to the *Entscheidungsproblem*"[7] in 1937, which became the basis of modern computing. In this connection, the first computer must have been a "computor".

Hence, mathematics embodied people's demands, intentions, and thoughts during its early development, and then it became an abstract and universal science. In this way, mathematics can be viewed as a common language underlying all other disciplines and thus can be used to contribute to other disciplines in science and engineering.

Three Elements in the Interdisciplinary Research Areas Between Mathematics and Other Scientific Disciplines

This section describes three typical elements of interdisciplinary studies between mathematics and other scientific disciplines: that is, applying existing theorems and techniques; clarifying the mechanisms of a target phenomenon using a mathematical model; finding mathematical structures embedded in the target phenomenon and thereby providing a new perspective of the phenomena.

The first element clarifies the mechanisms of observed phenomena; therefore, we identify adequate mathematical theorems to apply in solving problems. The key aspect of this element is the identification of the most appropriate mathematical language for each phenomenon. Therefore, mathematical techniques, comprehension of the phenomenon, and techniques for mathematically representing these phenomena are necessary.

Newton's discovery of differential and integral calculus, simply calculus, is the most profound and influential example of work in the second element of interdisciplinary research. Newton invented calculus in creating a mathematical model for the laws of motion. Similarly, Fourier invented Fourier analysis to clarify the mechanisms of complex spatiotemporal changes in states including periodic behaviors, such as waves and heat conduction. René Thom[8] invented catastrophe theory to clarify the mechanism of morphogenesis and various complex changes of states appearing in human society.

One of my own studies focuses on the functional differentiation of the brain. By constructing a new framework of constrained variational principles for the developmental process of the brain from fetus to adult, we applied a theory of self-organization with constraints to the network of dynamical systems including reservoir computers. We then introduced a genetic algorithm to change dynamical systems to adapt to given constraints in this framework.[9] Our target was a (random) network of a family of dynamical systems. We succeeded in realizing various types of functional differentiations, such as the formation of functional modules, the emergence of neuron-like and glial cell-like functional units, and a sensation-specificity of neurons.[10,11]

Finally, examples of the third element of interdisciplinary research can typically be found in life sciences, including medical science. Mathematics may traditionally be considered an inadequate language for the life sciences in Gelfand's sense. Nevertheless, following the development of data science and artificial intelligence, the use of mathematics has become crucial and even decisive in analyzing the complex phenomena of living systems.

Therefore, life science is becoming a part of mathematical science by force of circumstance.[12,13] My own research clarifying the functional meaning of dynamic associative memory belongs to this class. Milnor-like attractors have been extracted in the nonequilibrium neural networks for associative memories, leading to the new notion of chaotic itinerancy, which plays a crucial role in the dynamic association of memories, that is, the formation of episodic memories.[14] Here, chaotic itinerancy provides different dynamics from conventional attractor dynamics. This dynamic transition is consistent with the experimental findings of Walter Freeman and his colleagues[15] in olfactory bulbs during olfaction learning. In addition, Cantor sets and iterated function systems in mathematical models for the hippocampus were extracted and verified by a slice experiment of the rat's hippocampus.[16,17] It is crucial to identify the embedded dynamical systems in complex experimental data and even in massive data obtained by computer simulations of a mathematical model for both understanding complex phenomena and providing new insights into those phenomena.

Here, I have discussed three elements of applied mathematics in interdisciplinary research. I hope that young applied mathematicians will identify their own methods for mathematical applications as an adequate language for other scientific disciplines, which will thereby contribute to mathematics itself and also to society.

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Kenji Kajiwara

Kyushu University, Japan

The Challenge of Developing Mathematics for Industry

Fostering the Diversity and Creativity of Mathematics in Japan

Among the four main islands of Japan, Kyushu is the westernmost, with an area of approximately 37,000 km^2 and a population of 12.6 million. Fukuoka, where Kyushu University is located, is the largest city in Kyushu and is situated on its northern coast. In terms of geographical advantage, Fukuoka has long been a gateway to overseas countries and a prosperous trading city known as "Hakata". According to the Chinese history *The Book of the Later Han*, it was already known as the Kingdom of "Na" in the 1st century AD, and its envoy received a gold seal from the Emperor of China. Miraculously, the gold seal was unearthed about 240 years ago and is preserved in the Fukuoka City Museum. Incidentally, the name of the main campus of Kyushu University, the Ito Campus, originates from the Kingdom of "Ito", which existed in this region in the same era as the Kingdom of Na.

Japan was a loose union of small, regional states since that early period, and strong overall governance was rarely practiced until modern times. Japan has been an island nation, overcoming the threat of invasion on several occasions, and was influenced to some extent by the powerful civilization and culture of China but kept its distance from it. Each region developed its own culture and society. For example, *ramen*: Each area has developed its own, unique *ramen*, which competes for popularity. Sweets, for another example: Each region and town has unique confectioneries that it is proud of and tries its best to market. Traveling around Japan, you can enjoy a great variety of sweets at airports, train stations, and expressway service areas. In Fukuoka, in the north of Kyushu, *Torimon* made from wheat and bean paste is popular, while in Kagoshima, in the south, *Karukan* made from yams is popular. Both have a delicate taste but are completely different. In the past 150 years, the concentration of the Japanese population in Tokyo has increased, and many local products have been brought together in Tokyo, leading to a loss of understanding of the diversity of Japan. This booklet is called *Tokyo Intelligencer*, but you cannot understand Japan by looking at Tokyo alone. Please take this opportunity to travel around the countryside. You can enjoy a great variety of delicious food in each region.

This diversity in Japan can also be seen in mathematics. The Institute of Mathematics for Industry (IMI) at Kyushu University conducts mathematical research that is somewhat different from that of other research institutions in mathematics. Mathematics for Industry (MfI) is an idea of a new area of research in mathematics that serves as a foundation of future technologies and which as well is valuable as mathematics in itself. MfI has been created through the challenge of responding to the demands of society and industry by reorganizing and merging pure and applied mathematics into flexible and versatile forms. The mission and activities of the IMI toward that end are introduced below.

Mission 1: Promotion of Research and Collaboration with Industry and Diverse Scientific Fields

Based on the idea of MfI, our first mission is to contribute to society by identifying research problems from real problems and constructing new mathematics to help solve those problems. In fact, in FY 2021 we carried out 25 joint research projects with industry. Our aim is not just to solve mathematical problems arising in industry, but also to meet the demands of industry and complete social implementation. For example, the IMI has been involved in the smartification of a pharmaceutical company's factory from the planning stage and has introduced both classical and quantum computers and developed a state-of-the-art mathematical algorithm to run on them. In December 2022, the Division of Fujitsu Mathematical Modelling for Decision Making was established with the support of the global IT company Fujitsu, and systematic joint research is under way.

More importantly, mathematics generated through such activities should be developed into attractive mathematics for researchers to work on and develop a new research area. This is the most distinctive and important mission of the IMI. Toward this end, the first priority of the IMI is to promote deep mathematical research that is both fundamental and cutting-edge. The IMI therefore includes researchers in pure mathematics and promotes the active involvement of mathematics researchers with various orientations, both pure and applied, in cross-disciplinary and industry–academia collaborations. Whether the problems to be solved are within or outside of mathematics, however, mathematics is nothing but mathematics, and it makes little sense to distinguish between different types of problems.

The IMI will conduct the “Mathematics for Industry Platform”, a network spread over Japan together with cooperative mathematics institutes for matching the needs of industry and various scientific fields, and the seeds of mathematics with the support of the Ministry of Education, Culture, Sports, Science and Technology (MEXT) of Japan. It will provide a one-stop service for those who need the involvement of mathematics in their research and development.

Mission 2: Joint Use Research

Another important mission of the IMI is to support the development of emerging mathematics that could become the seeds of industrial and applied mathematics. The IMI has been selected as a Joint Usage / Research Center by MEXT, and conducts 20 – 25 joint research projects such as workshops, short-term joint research, and short-term fellowships every year through open calls for proposals. The unique feature is that participants from companies are required in principle to attend, thus shaping the direction of the development of mathematics toward an awareness of industry and society. Recently, the IMI has been conducting one or two international research projects per year.

Mission 3: Human Resource Development

There is a strong need worldwide for people with a high level of mathematical knowledge that can be used not only in academia but also in industry and society. To achieve a high degree of mathematical knowledge, it is necessary to study mathematics in depth for a certain period of time, but it is not easy to balance this requirement with the development of a vision of mathematics for various scientific fields and industry. The IMI has prepared several mechanisms to bring students majoring in mathematics into contact with other fields and industry. One is the IMI Colloquium, where representatives of companies give lectures on the state of mathematics in their fields. There is also a Study Group Workshop, where students work on problems presented by companies for a week. A long-term internship, specially designed for Ph.D. students, is another means that Kyushu University has pioneered in Japan for mathematics students. To make those mechanisms more systematic and practical, the IMI runs the Joint Graduate School of Mathematics for Innovation in collaboration with the Graduate School of Mathematics, the Graduate School of Information Science and Electrical Engineering, and the Graduate School of Economics, and implements the WISE Program (Doctoral Program for World-leading Innovative & Smart Education) of MEXT. The program requires students to undertake joint research in laboratories in diverse fields and long-term internships and aims to nurture human resources capable of using mathematics to make innovations in other fields and industry.

Mission 4: International Collaborations

There is a strong pioneering community of industrial and applied mathematics in the USA, the Society of Industrial and Applied Mathematics (SIAM); and in Europe, the European Consortium of Mathematics for Industry (ECMI), leading the world community. The IMI has therefore set its sights on international collaborative activities with an emphasis on the Asia Pacific region. Toward this end, it has established the Asia Pacific Consortium of Mathematics for Industry (APCMfI) in cooperation with researchers in Australia, New Zealand, and East and Southeast Asian countries. The APCMfI organizes an annual international conference, the Forum "Math-for-Industry", on a rotating basis among the member institutions. It has also begun to raise its level of international recognition by organizing mini-symposiums at SIAM and ECMI events this year. To promote collaborations between Australia and Japan, where the community is relatively large, the IMI has established the Australia Branch at La Trobe University in Melbourne. The IMI Australia Branch is staffed by a faculty member employed jointly by La Trobe University and carries out a variety of joint activities.

In the week following ICIAM 2023, August 29th – September 1st, the IMI will host the Forum "Math-for-Industry" 2023—MfI2.0—as a satellite meeting of ICIAM2023. It will bring together a wide range of talks on hot topics in MfI from Japan and around the world, providing an opportunity to share the state-of-the-art and future perspectives of MfI at a location only 2 hours by plane from Tokyo, with over 60 daily flights, and then only 20 minutes by subway from the airport to the venue. Partial attendance is welcome. For details, please refer to the webpage `http://apcmfi.org/fmfi2023/index.html`. On this occasion, why not visit the historic city of Fukuoka, for fascinating food and mathematics?

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Double Exponential Transformation: A Quick Review of a Japanese Tradition

Abstract

This article is a short introduction to numerical methods using the double exponential (DE) transformation, such as tanh-sinh quadrature and DE-Sinc approximation. The DE-based methods for numerical computation have been developed intensively in Japan and the objective of this article is to describe their history in addition to the underlying mathematical ideas.

Keywords: Double exponential transformation, DE integration formula, tanh-sinh quadrature, DE-Sinc method.

1 Introduction

The double exponential (DE) transformation is a generic name of variable transformations (changes of variables) used effectively in numerical computation on analytic functions, such as numerical quadrature and function approximation. A typical DE transformation is a change of variable x to another variable t by $x = \phi(t)$ with the function

$$\phi(t) = \tanh\left(\frac{\pi}{2}\sinh t\right).$$

The term "double exponential" refers to the property that the derivative

$$\phi'(t) = \frac{\frac{\pi}{2}\cosh t}{\cosh^2(\frac{\pi}{2}\sinh t)}$$

decays double exponentially

$$\phi'(t) \approx \exp\left(-\frac{\pi}{2}\exp|t|\right) \tag{1}$$

as $|t| \to \infty$.

This article is a short introduction to numerical methods using DE transformations such as the double exponential formula (tanh-sinh quadrature) for numerical integration and the DE-Sinc method for function approximation.

The DE-based methods for numerical computation have been developed intensively in Japan [5, 7, 34, 38], and a workshop titled "Thirty Years of the Double Exponential Transforms" was held at RIMS (Research Institute for Mathematical Sciences, Kyoto University) on September 1–3, 2004 [14]. The objective of this article is to describe the history of the development of the DE-based methods in addition to the underlying mathematical ideas.

This article is written in memory of Professors Masao Iri (President of Japan SIAM, 1996), Masatake Mori (President of Japan SIAM, 1998), and Masaaki Sugihara (Vice President of Japan SIAM, 2008).

2 DE Formula for Numerical Integration

The DE formula for numerical integration invented by Hidetosi Takahasi and Masatake Mori [37] was first presented at the RIMS workshop "Studies on Numerical Algorithms," held on October 31–November 2, 1972. The celebrated term "double exponential formula" was proposed there, as we can see in the proceedings paper [36].

2.1 Quadrature Formula

The DE formula was motivated by the fact that the trapezoidal rule is highly effective for integrals over the infinite interval $(-\infty, +\infty)$. For an integral

$$I = \int_{-1}^{1} f(x)\,\mathrm{d}x,$$

for example, we employ a change of variable $x = \phi(t)$ using some function $\phi(t)$ satisfying $\phi(-\infty) = -1$ and $\phi(+\infty) = 1$, and apply the trapezoidal rule to the transformed integral

$$I = \int_{-\infty}^{+\infty} f(\phi(t))\phi'(t)\,\mathrm{d}t,$$

to obtain an infinite sum of discretization

$$I_h = h \sum_{k=-\infty}^{\infty} f(\phi(kh))\phi'(kh). \qquad (2)$$

A finite-term approximation to this infinite sum results in an integration formula

$$I_h^{(N)} = h \sum_{k=-N}^{N} f(\phi(kh))\phi'(kh). \qquad (3)$$

Such combination of the trapezoidal rule with a change of variables was conceived by several authors [2, 24, 25, 35] around 1970.

The error $I - I_h^{(N)}$ of the formula (3) consists of two parts, the error $E_{\mathrm{D}} \equiv I - I_h$ incurred by discretization (2) and the error $E_{\mathrm{T}} \equiv I_h - I_h^{(N)}$ caused by truncation of an infinite sum I_h to a finite sum $I_h^{(N)}$.

The major findings of Takahasi and Mori consisted of two ingredients. The first was that the double exponential decay of the transformed integrand $f(\phi(t))\phi'(t)$ achieves the optimal balance (or trade-off) between the discretization error E_{D} and the truncation error E_{T}. The second finding was that a concrete choice of

$$\phi(t) = \tanh\left(\frac{\pi}{2}\sinh t\right) \qquad (4)$$

is suitable for this purpose thanks to the double exponential decay shown in (1). With this particular function $\phi(t)$ the formula (3) reads

$$I_h^{(N)} = h \sum_{k=-N}^{N} f\left(\tanh\left(\frac{\pi}{2}\sinh(kh)\right)\right) \times \frac{(\pi/2)\cosh(kh)}{\cosh^2((\pi/2)\sinh(kh))},$$

which is sometimes called "tanh-sinh quadrature." The error of this formula is estimated roughly as

$$\left|I - I_h^{(N)}\right| \approx \exp(-CN/\log N) \qquad (5)$$

with some $C > 0$. The DE formula has an additional feature that it is robust against end-point singularities of integrands.

The idea of the DE formula can be applied to integrals over other types of intervals of integration. For example,

$$I = \int_{0}^{+\infty} f(x)\,\mathrm{d}x, \quad x = \exp\left(\frac{\pi}{2}\sinh t\right), \qquad (6)$$

$$I = \int_{-\infty}^{+\infty} f(x)\,\mathrm{d}x, \quad x = \sinh\left(\frac{\pi}{2}\sinh t\right). \qquad (7)$$

Such formulas are also referred to as the double exponential formula. The DE formula is available in Mathematica (NIntegrate), Python library SymPy, Python library mpmath, C++ library Boost, Haskell package integration, etc.

2.2 Optimality

Optimality of the DE transformation (4) was discussed

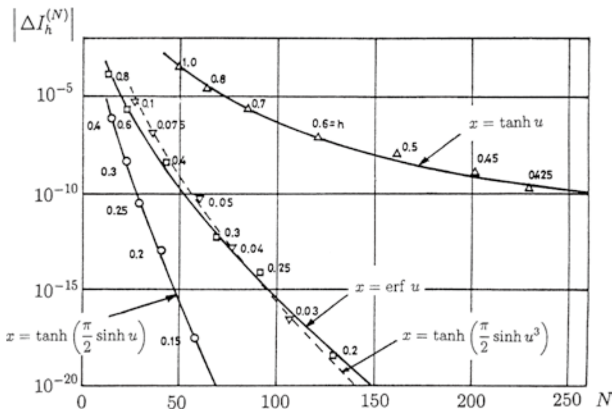

Figure 1.

Comparison of the efficiency of several variable transformations for the integral $\int_{-1}^{1} \mathrm{d}x/\{(x-2)(1-x)^{1/4}(1+x)^{3/4}\}$; taken from Mori [5, Fig. 4] with permission from the European Mathematical Society; u and N in the figure correspond, respectively, to t and $2N+1$ in the present notation.

already by Takahasi and Mori [37]. Numerical examples also support its optimality. Figure1 (taken from [5]) shows the comparison of the DE transformation (4) against other transformations

$$\phi(t) = \tanh t,$$

$$\phi(t) = \tanh\left(\frac{\pi}{2}\sinh t^3\right),$$

$$\phi(t) = \operatorname{erf}(t) = \frac{2}{\sqrt{\pi}}\int_0^t \exp(-s^2)\,\mathrm{d}s$$

for $\int_{-1}^{1} \mathrm{d}x/\{(x-2)(1-x)^{1/4}(1+x)^{3/4}\}$ that has integrable singularities at both ends of the interval of integration. The DE formula converges much faster than others. It is known that the tanh-rule (using $\phi(t) = \tanh t$) has the (rough) convergence rate $\exp(-C\sqrt{N})$, in contrast to $\exp(-CN/\log N)$ in (5) of the DE formula.

The optimality argument of [37], based on complex function theory, was convincing enough for the majority of scientists and engineers, but not perfectly satisfactory for theoreticians. Rigorous mathematical argument for optimality of the DE formula was addressed by Masaaki Sugihara [28, 29, 30] in the 1980s and 1990s in a manner comparable to Stenger's framework [26] for optimality of the tanh rule. It is shown in [30] (also [42]) that the DE formula is optimal with respect to a certain class (Hardy space) of integrand functions.

In principle, for each class of integrand functions we may be able to find an optimal quadrature formula, and the optimal formula naturally depends on our choice of the admissible class of integrands. Thus the optimality of a quadrature formula is only relative. However, it was shown by Sugihara that no nontrivial class of integrand functions exists that admits a quadrature formula with smaller errors than the DE formula. We can interpret this fact as the absolute optimality of the DE formula.

2.3 Fourier-Type Integrals

For Fourier-type integrals such as

$$I = \int_0^{+\infty} f_1(x)\sin x\,\mathrm{d}x,$$

the DE formula like (6) is not very successful. To cope with Fourier-type integrals, a novel technique, in the spirit of DE transformation, was proposed by Ooura and Mori [22, 23]. In [22] they proposed to use

$$\phi(t) = \frac{t}{1-\exp(-K\sinh t)}$$

$(K > 0)$, which maps $(-\infty, +\infty)$ to $(0, +\infty)$ in such a way that (i) $\phi'(t) \to 0$ double exponentially as $t \to -\infty$ and (ii) $\phi(t) \to t$ double exponentially as $t \to +\infty$. The proposed formula changes the variable by $x = M\phi(t)$ to obtain

$$I = M\int_{-\infty}^{+\infty} f_1(M\phi(t))\sin(M\phi(t))\phi'(t)\,\mathrm{d}t,$$

to which the trapezoidal rule with equal mesh h is applied, where M and h are chosen to satisfy $Mh = \pi$. The transformed integrand decays double exponentially toward $t \to -\infty$ because of the factor $\phi'(t)$ and also toward $t \to +\infty$ because $M\phi(t)$ for $t = kh$ (sample point of the trapezoidal rule) tends double exponentially to $Mt = Mkh = k\pi$, at which sine function vanishes. Another (improved) transformation function

$$\phi(t) = \frac{t}{1-\exp(-2t-\alpha(1-\mathrm{e}^{-t})-\beta(\mathrm{e}^{t}-1))},$$

is given in [23], where the parameters are $\beta = 1/4$ and $\alpha = \beta/\sqrt{1+M\log(1+M)/(4\pi)}$.

2.4 IMT Rule

In 1969, prior to the DE formula, a remarkable quadrature formula was proposed by Masao Iri, Sigeiti Moriguti, and Yoshimitsu Takasawa [2]. The formula is known today as the "IMT rule," which name was introduced in [35] and used in [1].

For an integral $I = \int_0^1 f(x)\,\mathrm{d}x$ over $[0, 1]$, the IMT rule applies the trapezoidal rule to the integral $I = \int_0^1 f(\phi(t))\phi'(t)\,\mathrm{d}t$ resulting from the transformation by

$$\phi(t) = \frac{1}{Q}\int_0^t \exp\left[-\left(\frac{1}{\tau} + \frac{1}{1-\tau}\right)\right]\mathrm{d}\tau,$$

where $Q = \int_0^1 \exp\left[-\left(\frac{1}{\tau} + \frac{1}{1-\tau}\right)\right]\mathrm{d}\tau$ is a normalizing constant to render $\phi(1) = 1$.

The transformed integrand $g(t) = f(\phi(t))\phi'(t)$ has the property that all the derivatives $g^{(j)}(t)$ $(j = 1, 2, \ldots)$ vanish at $t = 0, 1$. By the Euler–Maclaurin formula, this indicates that the IMT rule should be highly accurate. Indeed, it was shown in [2] via a complex analytic method that the error of the IMT rule can be estimated roughly as $\exp(-C\sqrt{N})$, which is much better than N^{-4} of the Simpson rule, say, but not as good as $\exp(-CN/\log N)$ of the DE formula. Variants of the IMT rule have been proposed for possible improvement [4, 10, 21, 29], but it turned out that an IMT-type rule, transforming $\int_0^1 \mathrm{d}x$ to $\int_0^1 \mathrm{d}t$ rather than to $\int_{-\infty}^{+\infty} \mathrm{d}t$, cannot outperform the DE formula.

3 DE-Sinc Methods

Changing variables is also useful in the Sinc numerical methods. The book by Stenger [27] in 1993 describes this methodology to the full extent, focusing on single exponential (SE) transformations like $\phi(t) = \tanh(t/2)$. Use of the double exponential transformation in the Sinc numerical methods was initiated by Sugihara [31, 33] around 2000, with subsequent development mainly in Japan. Such numerical methods are often called the DE-Sinc methods. The subsequent results obtained in the first half of the 2000s are described in [5, 7, 34].

3.1 Sinc Approximation

The Sinc approximation of a function $f(x)$ over $(-\infty, \infty)$ is given by

$$f(x) \approx \sum_{k=-N}^{N} f(kh)S(k,h)(x), \qquad (8)$$

where $S(k,h)(x)$ is the so-called Sinc function defined by

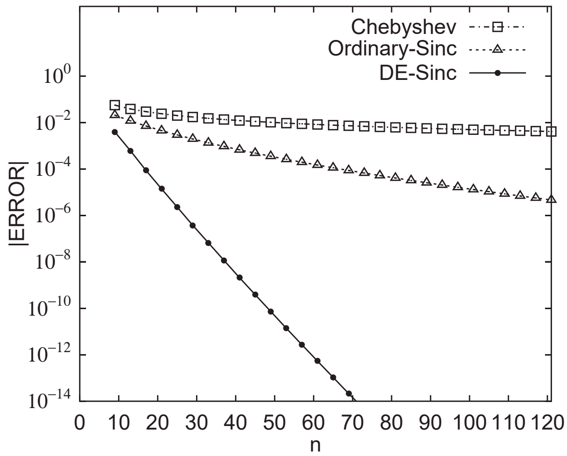

Figure 2.
Errors in the Sinc approximations for $x^{1/2}(1-x)^{3/4}$ using (9) and (10) and the Chebyshev interpolation. From Sugihara and Matsuo [34, Fig. 3] with permission from Elsevier; n in the figure corresponds to N in (8).

$$S(k,h)(x) = \frac{\sin[(\pi/h)(x-kh)]}{(\pi/h)(x-kh)}$$

and the step size h is chosen appropriately, depending on N. The technique of variable transformation $x = \phi(t)$ is also effective in this context. By applying the formula (8) to $f(\phi(t))$ we obtain

$$f(\phi(t)) \approx \sum_{k=-N}^{N} f(\phi(kh))S(k,h)(t),$$

or equivalently,

$$f(x) \approx \sum_{k=-N}^{N} f(\phi(kh))S(k,h)(\phi^{-1}(x)).$$

To approximate $f(x)$ over $[0, 1]$, for example, we choose

$$\phi(t) = \frac{1}{2}\tanh\frac{t}{2} + \frac{1}{2}, \qquad (9)$$

$$\phi(t) = \frac{1}{2}\tanh\left(\frac{\pi}{2}\sinh t\right) + \frac{1}{2}, \qquad (10)$$

etc. The methods using (9) and (10) are often called the SE- and DE-Sinc approximations, respectively. The error of the SE-Sinc approximation is roughly $\exp(-C\sqrt{N})$ and that of the DE-Sinc approximation is $\exp(-CN/\log N)$.

These approximation schemes are compared in Fig. 2 (taken from [34]) for function $f(x) = x^{1/2}(1-x)^{3/4}$ over $[0, 1]$. In Fig. 2, "Ordinary-Sinc" means the SE-Sinc approximation using (9), and the polynomial interpolation with the Chebyshev nodes is included for comparison.

Detailed theoretical analyses on the DE-Sinc method can be found in Sugihara [33] as well as Tanaka et al. [41] and Okayama et al. [16, 20]. An optimization technique is used to improve the DE-Sinc method in Tanaka and Sugihara [39].

3.2 Application to Other Problems

Once a function approximation scheme is at hand, we can apply it to a variety of numerical problems. Indeed this is also the case with the DE-Sinc approximation as follows.

- Indefinite integration by Muhammad and Mori [8], Tanaka et al. [40], and Okayama and Tanaka [19].
- Initial value problem of differential equations by Nurmuhammad et al. [11] and Okayama [15].
- Boundary value problem of differential equations by Sugihara [32], followed by Nurmuhammad et al. [12, 13] and Mori et al. [6].
- Volterra integral equation by Muhammad et al. [9] and Okayama et al. [18].
- Fredholm integral equation by Kobayashi et al. [3], Muhammad et al. [9], and Okayama et al. [17].

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Encyclopedic Dictionary of Mathematics Traces Numerical Analysis in Japan

1 Introduction

Numerical analysis means the mathematical discipline to design, analyse and evaluate numerical algorithms in continuous mathematics and therefore holds a significant place in applied mathematics. In its long history, several giants such as Issac Newton and Carl Friedrich Gauss remained as inventors of new algorithms. Thus the theory and practice of numerical analysis has an international nature as mathematics itself has. On the other hand, its study and development often exhibits individuality that reflects the uniqueness of each country, because they are carried out by scientists and practitioners in that particular country. On this note, the author tries to describe the study and development of numerical analysis in the modern era in Japan through the publication series—the Encyclopedic Dictionary of Mathematics (EDM) edited by the Mathematical Society of Japan (MSJ).

In its scientific sense, the study of and education in mathematics started in Japan only about one and half centuries ago. Since then, a rapid development can be observed but it was inclined to so-called pure mathematics. However, social and engineering developments in Japan required the application of mathematics and thus prompted development of applied mathematics. The MSJ was founded in 1877 as Tokyo-Sugaku-Kaisha (Tokyo Mathematics Society) with 55 members. Subsequently, it was expanded to Tokyo-Sugaku-Buturi-Kaisha (Tokyo Mathematical and Physical Society) and later to the Physico-Mathematical Society of Japan. It was split in 1946 into two separate societies: "The Mathematical Society of Japan" and "The Physical Society of Japan", and the MSJ as we have it today was established at that time. The Japan Society for Industrial and Applied Mathematics (JSIAM) was established in

April 1990 to support and collaborate with activities related to applied mathematics research, industry, and education in Japan. Taking this background into account, we investigate the historical developments of EDM by concentrating our focus on numerical analysis.

2 First to third editions

In the spring of 1947, only a year after its establishment, MSJ started the publication project of EDM and appointed an editorial board chaired by Shôkichi Iyanaga (1906 – 2006). In his foreword he explained the editorial policy as well as the guiding principle of EDM, which have been applied to the successive editions, too. Iyanaga adopted the "middle-size article" principle, that is, each article in EDM describes a certain concept or fact that is well-structured as an explanation. Also several areas of mathematics such as algebra, geometry and functional equations were established to select articles in some sections. Nearly 30 Japanese mathematicians organised an editorial board, selected articles to be described and asked many colleagues to provide descriptions in each article. It was a big project organised by MSJ and eventually published by Iwanami Shoten Publishers in Tokyo in the spring of 1954.

The first edition of EDM, which comprised 778 pages, contained no section or descriptive article on numerical analysis. Instead, among 64 articles in the applied mathematics section, several described topics in numerical analysis according to their contents. Those are:

> numerical calculation, interpolation, numerical differentiation, numerical integration, least-square method, numerical solution of linear equations, numerical solution of algebraic equations, numerical solution of transcendental equations, numerical solution of ordinary differential equations, numerical solution of partial differential equations, numerical solution of integral equations, relaxation methods, steepest descent method, W.K.B. method, graphical solution methods, and nomography.

We can see that classical topics in numerical analysis were covered in EDM, but at the same time we are aware this is a list from the 1950s. As usual in EDM, the names of article authors were not shown with specific articles. However, the lists of editorial board members as well as of article authors give a hint. Shigeiti Moriguti (1916 – 2002), who was on the editorial board, must have written several articles. Also Ayao Amemiya (1907 – 1977), Isao Imai (1914 – 2004), Kunihiko Kodaira (1915 – 1997) and Tosio Kato (1917 – 1999) possibly wrote several articles, as well. Many of these individuals can be said to have been physicists or applied physicists. This means that at that time physics and applied science had a close relationship with mathematics in its practical application.

In 1960 the supplementary edition of EDM was published, however, the descriptions in the applied mathematics section were not revised. In the spring of 1968 the second edition of EDM of 1140 pages was published. Iyanaga again served as the editorial chair and the numerical analysis section was established. It was a good development to place numerical analysis among the mathematical disciplines. Hiroshi Fujita (1928 –) and Masaya Yamaguti (1925 – 1998) served as the editorial members in charge of the section and many names of numerical analysts can be found in the list of authors. The following 19 articles were listed in the section.

> numerical calculation, interpolation, polynomial approximation, error analysis, numerical solution of linear equations, numerical computation of eigenvalues, numerical solution of algebraic equations, numerical integration, numerical solution of ordinary differential equations, numerical solution of partial differential equations, relaxation methods, P.L.K. method, W.K.B. method, steepest descent method, graphical solution methods, nomography, curve fitting, computer, and analogue computer.

Several featured expressions were found in the articles. The article "numerical calculation" carries the phrase "numerical analysis" and its implication is given. The article on "numerical solution of linear equations" mentions the conjugate gradient (CG) method, but questions its practical value. The article "numerical solution of ordinary differential equations" stresses discrete variable methods. The article "numerical

Figure 2.1:
Shôkichi Iyanaga (top); Shigeiti Moriguti (bottom left); Tosio Kato (bottom right)

Figure 2.2:
EDM supplemented 1st edition. (a used copy)

solution of partial differential equations" refers to Ritz–Galerkin and finite difference methods, but not to the finite element method. Also, we can note that mathematical descriptions of digital computers appear to reflect the scientific developments of the 1960s.

In the autumn of 1985 the third edition of EDM, with 1609 pages, was published. This time Kiyosi Itô (1915 – 2008) acted as the editorial chair and a total of 21 sections were established. The section "numerical analysis" was continued, but the mathematical contents of numerical analysis were also included in articles of the sections "computer mathematics and combinatorial mathematics" and "mathematics of programming". M. Yamaguti again served as the editorial member in charge of the numerical analysis section and was strongly assisted by Teruo Ushijima (1939 –). The articles in the section were rearranged and their description was updated. The 11 articles in the section are on:

> numerical calculation, interpolation, error analysis, numerical solution of linear equations, numerical computation of eigenvalues, numerical solution of algebraic equations, numerical integration, numerical solution of ordinary differential equations, numerical solution of partial differential equations, analogue computation, and evaluation of

Figure 2.3:
Kiyosi Itô (top); Hiroshi Fujita (bottom left); Masaya Yamaguti (bottom right)

functions.

The article on "numerical solution of linear equations" mentions the incomplete Cholesky conjugate gradient (ICCG) mathod and singular value decomposition. The article on "numerical computation of eigenvalues" is updated to present Lanczos and QR methods and the Hessenberg form. The double exponential (DE) formula and the adaptive quadrature are described in the article on "numerical integration". The article on "numerical solution of ordinary differential equations" refers to the stiff problem, whereas the article on "numerical solution of partial differential equations" describes the finite element method in detail. The fast Fourier transform (FFT) is referred to in the article on "evaluation of functions".

The third edition also has an English translation. Four volumes of the *Encyclopedic Dictionary of Mathematics* were published by the MIT Press in 1987.

3 Fourth edition

After 20 years had passed since the publication of the third edition of EDM, MSJ recognised that mathematics generally had attained a great progress and its branches had deepened their mutual cooperation. At the same time, other scientific disciplines as well as human society began to apply mathematics more seriously. To meet such developments and requirements, in 2001 MSJ decided to start the project for the fourth edition of EDM, and Akio Hattori (1929 – 2013) was appointed the editorial chair. The editorial principle was kept similar to that of the previous editions, but some modernisation was carried out. The number of sections increased to 23, from 21 in the previous edition. The editorial board consisted of core members and expert members. Each core member was in charge of several sections and selected the articles in those sections, while several expert members were in charge of single sections to select and coordinate each article author. At the end of 2006, all the efforts of the people who were devoted to the project came to fruition and the fourth edition of 1,976 pages was published with a CD-ROM attached. The CD-ROM also included the digitised contents of the third edition for comparison's sake.

The present author was asked to join the editorial board as a core member and to be in charge of the sections "numerical analysis", "discrete and combinatorial mathematics", "mathematics in informatics" and "optimization theory". Colleagues Masaaki Sugihara,

Figure 3.1:
Akio HATTORI

Figure 3.2:
EDM 4th edition; outer case (left), cover (right)

Masahisa TABATA, Mitsuhiro NAKAO, Hiroshi KONNO and Kazuo MUROTA greatly supported the editorial work of numerical analysis as expert members. The 11 articles in the numerical analysis section are listed below with a short comment.

numerical analysis explaining "What is numerical analysis?"; error analysis is included.

numerical solution of linear equations mentioning modern conjugate-gradient (CG) methods, the multi-grid method and singular value decomposition.

numerical solutions of nonlinear equations modern Newton iteration, the Kantorovich theorem and the homotopy method are given.

numerical computation of eigenvalues QR, Arnoldi and Lanczos methods are given together with Schur decomposition.

numerical integration explaining DE formula, multi-dimensional integration and the Monte Carlo method.

numerical solution of ordinary differential equations rooted trees and stability analysis are described.

numerical solution of partial differential equations explaining boundary-element method (BEM) and spectral methods.

finite difference method for partial differential equations.

finite element method for partial differential equations.

evaluation of functions explaining interpolation, errors, acceleration, CORDIC and FFT.

self-validating methods based on the interval analysis.

We can observe that an updated explanation at the beginning of the twenty-first century was given in each article. The present author believes that this is a reflection of longtime activities of research in Japan for numerical analysis as well as of the results of their intercommunication with practitioners. Of course international scientific exchange made a great impact, too.

4 Future

The reader might wonder what any relevant publication of EDM would be like after the fourth edition. As far as the author is aware, there is no such plan. One reason might be the great difficulty organising an editorial team as well as a reliable group of article authors for the EDM project. However, the major reason would be the rapid development of the internet as well as the spread of information on the Web. Printed books are becoming less popular. Now everybody can access Web pages, which provide enormous amounts of information including on mathematics and numerical analysis. For example, try inputting the string of words "numerical solution of linear equations" into the Web browser of your computer and hit the search key. You can find more than 100 mega hits for the string. The first natural embarrassment is the question, "Which fits my purpose?" This suggests that a reliable and systematic source of information is still greatly needed, particularly for scientific contents. Thus the author believes a publication like the EDM is still required to perform scientifically tenable study. The development of digital technology can rather contribute to the demand. Suppose we have a new digital EDM. Then, cross references between articles must be very easy and we can obtain a systematic view of each discipline. Moreover, as described in the previous sections, by tracing the changes of a single article between the editions we can understand its historical development and might obtain an idea for further progress. A proverb says that to know new things, learn by studying the old. The author strongly hopes for such a new edition in the near future.

Note: A short description of the EDM series can be found in *Wikipedia*:

`https://en.wikipedia.org/wiki/Encyclopedic_Dictionary_of_Mathematics`

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Recollections on my 60+ years on mathematical sciences[1]

Professor Hiroshi Fujita, the former president of the Japan Society for Industrial and Applied Mathematics (JSIAM) received the Kodaira Kunihiko Prize in 2019. On this occasion, the second author, the then president of JSIAM and a former pupil of the first author, planned the following article, composed of several questions posed by the second author and responses to them by the first author.

Q1. Please describe your early life until graduation from the University of Tokyo.

I, Hiroshi Fujita, was born on December 7th, 1928, in Osaka City. My father, Yoshisato, was originally from Ehime Prefecture and was working as an English teacher. After graduating from Naniwa High School of the old system of education, I was enrolled in the University of Tokyo on April 1st, 1948. I well remember an event when I took the entrance examination. In a seat two rows behind mine was sitting Masatoshi Koshiba, on whom the Nobel Prize in Physics was later bestowed. After having handed examination papers to examinees, the examiner asked, "Are there any questions?" Koshiba raised his hand and asked, "May I smoke?"

My school records in the Physics Department were mixed. Experimental subjects were "Excellent" but mathematical physics was "Good". At the time I was studying at the University of Tokyo, students were expected to graduate in three years. However, it took four years for me. This was because I was unable to concentrate on learning: My father was killed by one of the bombings of Osaka by B29 bombers during the Second World War, and I had to earn living expenses by myself. Students in the last year of the university were to choose either a physics

1 This part is based on an article first published in *Ouyousuuri*, vol. 30, issue 1 (2020), pp. 38–42. https://doi.org/10.11540/bjsiam.30.1_38

Tosio Kato and his students in the 1950s. Seated, from the left: Tosio Kato, Teruo Ikebe, Hiroshi Fujita, Yoshimoto Nakata. Standing: Shige-Toshi Kuroda.

experiment or a reading seminar of theoretical texts. Together with two of my classmates I chose the reading seminar. The supervisor was Takahiko Yamanouchi, a senior professor famous for the applications of group theory to quantum mechanics. He told us: "Recently such a new tool as distribution was invented. It will be important in the study of mathematical physics. To study the theory of distribution it will be necessary to be familiar with general topology." Thus, we began reading *Topologie Générale* by Bourbaki. Professor Yamanouchi was surely a mathematical physicist of the first class, but the problems (exercises of Bourbaki) were quite difficult even for him. Then Tosio Kato was promoted to assistant professor and Professor Yamanouchi ordered us to study under Kato's supervision. This was in the summer of 1951.

Q2. How was life in graduate school?

I continued my study under Kato's guidance. I was awarded a scholarship and my living circumstances were improved very much. In addition to the learning and research, I acted as an assistant in Kato's laboratory. Also, I taught at Kaisei High School as a part-time teacher. In those years, it was customary for graduate students to defend their Doctor's degree by submitting a thesis. It may sound curious to current readers, but there were those graduate students who were not interested in Doctor's degrees. For instance, the famous scholar Hidetosi Takahasi defended his thesis only at the stage of his promotion to assistant professor (which was equivalent to a present associate professor) at the strong urging of the department leaders.

Actually, during the starting few years of my research, I didn't care much about the thesis. The first group of my papers was concerned with approximate numerical methods based on Kato's T^*T theory. I carried out numerical computation by a mechanical analogue computer called Tiger. The T^*T theory is acclimatable to the finite element method (FEM), hybrid FEM in particular, and later in the 1970s led me to the study of FEM. It also motivated my collaboration with engineers. In addition, based on Kato's advice, I studied carefully the finite difference method, too. His advice was something like this: The era of applied mathematics using a computer is emerging, and you should be prepared for it. Since you are studying mathematics in the Physics Department, it won't be easy to find an academic job. Therefore, you should arm yourself with the weapons of numerical methods in the use of computers.

I was employed as a research assistant in the Physics Department in the summer of 1956. In the spring of 1957, I married Yoshiko. Kato told me that it was about time to submit my thesis, when I encountered the paper by Kiselev and Ladyzhenskaya (called "the K-L paper" for short) published in *Izvestiya*. This was the beginning of my research on the Navier–Stokes equations. I learned of the works by J. Leray and E. Hopf from the K-L paper. In those days, books and journals from Western countries were expensive for Japanese scholars, whereas Russian books and journals were cheap. Therefore, Kato's students often read books and journals in Russian with much effort. The K-L paper used a Galerkin method suitable for a time-dependent system in order to prove the existence of the solution of the initial-boundary value problem for nonstationary flows. It then naturally occurred to me that the stationary solutions could be transparently constructed by some sort of Galerkin method. This idea led me to my thesis. From this experience, I learned the lesson that a good approximation method was a good method of theoretical analysis. Near the completion of the paper, I learned of Robert Finn's paper on exterior flows that approach a uniform flow at infinity. I therefore poured my energy into deriving a result no less important than his.

Q3. Please tell us of your experience with digital computers in those early days.

It was when I was an assistant in the Physics Department that I first used a digital computer. Around that time, Eiichi Goto invented a parametron, a circuit element, by which Goto's teacher, Professor Takahasi, and his co-workers built the PC-1 (parametron computer 1), whose memory was a mere 8-bit 500 words. Nevertheless, the time-marching finite difference scheme yielded a solution of the initial-boundary value problem of the heat equation in one dimension. Professor Takahasi, a brilliant physicist, also had an insight into numerical analysis and computational method. It is well known that Masatake Mori was inspired by Takahasi to discover their celebrated double exponential formula for quadrature. Eiichi Goto also had unusually keen insight into numerical analysis.

Q4. Please describe for us your research experience in the USA in the 1960s.

I first visited the USA in September1962. I went to Stanford University and stayed there for a year and a half with Professor Robert Finn. From San Francisco airport to Stanford, I saw many cars running at high speed on a freeway for the first time in my life. I was impressed by the spectacular view. My title at Stanford was research associate, but a year later I was assigned to be a lecturer to teach. The salary was about seven times more than the one in Tokyo. At that time, I was working at the University of Tokyo as a lecturer in the Faculty of Engineering, where the teaching load in mathematics and mechanics was heavy, and an 18-month leave was considered to be long. On the other hand, the Faculty of Science imposed a lighter teaching load. During my stay, Stanford University paid my salary—a considerable difference from the current situation both in Japan and in America. After finishing the term in Stanford, I came back to the University of Tokyo and was soon promoted to full professor in the Mathematics Department of the Faculty of Science. In 1967, I visited the Courant Institute of Mathematical Science at New York University and stayed there for one year as visiting faculty. It was L. Nirenberg who invited me to Courant as he was seriously interested in my result for the blow-up problem.

Q5. Please tell something about the blow-up problem.

It would be my paper on the blow-up problem that came to be cited most frequently among my papers. It was only many years later that this work became widely recognized as a valuable achievement. On the other hand, I was mostly satisfied with my proposal of such a typical problem, which focuses on the interaction between the space-dimension and the degree of nonlinearity in the Cauchy problem for the typical nonlinear heat equation, which later became known as the Fujita equation.

My motivation in looking for such a model equation was as follows: The difficulty of three-dimensional Navier–Stokes equations were well known, while the problem had been solved in two dimensions. If going straight is difficult, you should go around the obstacle and you should extract the essence of the relation between nonlinearity and spatial dimensions. I must add that my thoughts during my derivation of the Fujita equation benefitted from the planning of physical experiments, where you set up a typical event; that is, an event in the circumstances where the essence of the phenomena should be revealed plainly, and the characteristics and mechanism of the problem in question become conspicuous. According to Professor Kunihiko Kodaira, mathematical phenomena do exist just as physical phenomena do, and mathematics is a science that tries to understand the mathematical phenomena. The proposal of the Fujita equation was, I believe, pioneering in the sense that it was invented to extract the essence of the mathematical phenomena involved in the blow-up problems. This desire of mine was better understood by foreign scholars than Japanese ones except for Eiji Yanagida and Noriko Mizoguchi. By analyzing the Fujita equation, I happily discovered what is called the Fujita Exponent—the pictures of blow-up changes drastically according to the degree that the nonlinearity is greater or less than the exponent. This result was predicted neither physically nor numerically. It was a surprise that, for fixed spatial dimensions, *all the positive solutions blow up* if the degree of nonlinearity is *smaller* than the Fujita Exponent.

Q6. How was your career from the Physics Department through the Faculty of Engineering to the Mathematics Department?

Right after obtaining my doctorate, I was promoted to lecturer in July 1960, in the Applied Physics Department of the Faculty of Engineering of the University of Tokyo. Tetsuro Inui invited me there. I spent six years, during which I became acquainted with younger friends Hideo Kawarada, Masatake Mori, and Makoto Natori, who later became leaders in the applied mathematics community of Japan, and I enjoyed the role of being the leader among them both in studying mathematics and in playing mah-jong. It was not long after my return from USA that I was invited to the Mathematics Department to become a professor. I was advised by Professor Kyuichirō Washizu, who was a fervent admirer of Kato's result on calculus of variations, not to move, saying, "The Faculty of Engineering treasures older people, while importance is attached more to younger people than older ones in the Faculty of Science. You are getting older, and I do not understand why you are moving to the Mathematics Department." But I did not follow his advice. When two mathematics professors, Kōsaku Yosida and Yukiyoshi Kawada, came to my office and enthusiastically urged me to come to the Mathematics Department, I was very touched by their personalities: Yosida's manly enthusiasm and Kawada's gentle thoughtfulness.

Q7. Please tell what you know about the establishment of the Japan Society of Industrial and Applied Mathematics (JSIAM).

It was Masatake Mori who played the most effective leading role in the establishment of JSIAM. I may have made some founding contribution to develop Tokyo–Kyoto collaboration in applied mathematics through my close association with Professor Masaya Yamaguti of Kyoto University prior to the start of JSIAM. During those years, I held a parallel appointment as a professor at the Research Institute for Mathematical Sciences, Kyoto University. Taking advantage of this, I established a friendly collaboration with him and set up a coalition between applied mathematics communities of the University of Tokyo and Kyoto University. It went well perhaps because both of us worked in mathematics departments but also had experienced working in faculties of engineering. As for the University of Tokyo, it might have been significant for the progress of applied mathematics that Masao Iri was then the Dean of the Faculty of Engineering of the University of Tokyo and I was the Dean of the Faculty of Science. As I stated in the symposium just before the founding of JSIAM, while paying respect to the systematic construction by rigorous proofs on the one hand, there should be manifold and multi-layered rules for judgments (for truth). For that matter, we should look at how consensus is formed by experiments in the physics community or how a court draws conclusions from collected evidence. In systematization by theory, drawing a whole from axioms, used in traditional mathematics, should not be the only doctrine; we should also utilize the ad-hoc understanding or follow the experience-based induction in order to attain a practical goal.

Q8. What was the beginning of the Japan–China seminar on numerical analysis?

Its beginning was my conversation in 1981 with Hua Lougeng, who was then the president of the Chinese

Mathematical Society. I visited China as the president of the Mathematical Society of Japan not long after the end of the Cultural Revolution, with the financial support of JSPS, the Japan Society for the Promotion of Science. After discussion with Hua Lougeng, we decided to resume academic exchanges in mathematics between the two societies. A thematic seminar was also set forth. Professor Feng Kang in the Chinese Academy of Science was in charge of it, and the first seminar, presided over by Professor Shi Zhong-ci, was held in Beijing in 1992. Thereafter the seminar was held every two years in China and Japan in turn. The second seminar was held at the University of Electro-Communications in Tokyo, for which Teruo Ushijima played a vital role. Nowadays joint papers by mathematicians in China and Japan are not uncommon, and we see many Chinese mathematicians teaching in Japanese universities. However, in the 1980s and 1990s, it was not an easy matter to support such seminars or symposiums. Hosts of the successive seminars must have spent a lot of effort for the success of the seminars. It was later consolidated with the China–Korea seminar, and Norikazu Saito is now in charge of it. I really hope that this project will endure against the political wind and continue contributing to the progress of applied mathematics in the new century.

Q9. What do you think of the new phase of mathematics education and/or mathematical sciences?

I was the president of the Mathematics Education Society of Japan in 2003–2008 and represented Japan in the International Commission on Mathematical Instruction in 1984–1994. Thus, I made a profound commitment to mathematics education. In the applied mathematics aspect of education, I was quite positive from the beginning about introducing computers in secondary schools. Or, I am proud to say that I played a leading role in the matter. When MEXT's Curriculum Guidelines for High Schools were revised in 1989, I was officially involved. Around that time I wrote that, "It will help foster literacy in mathematics to use a computer or a calculator to familiarize students with mathematical concepts and facts by manual movements. Further, we can expect it to help those students who are unable to follow the traditional definition-theorem-proof system to grasp sophisticated concepts." Computers are not easily adopted for examinations. Also, those who were willing to use such tools as computers were still a minority in Japan. The use of computers in high schools was not welcomed. I remember a good mathematician having expertise in mathematics education once said that people would soon use computers in the same way that we use telephones, i.e., without special knowledge. This was certainly an irresponsible remark. As time went on, many countries introduced computer-oriented education, and the gap between those countries and Japan has widened seriously. Japan must try very hard to catch up with them.

I would like to add that "mathematics literacy" is a term that I introduced in 1969 for the first time ever in the world. I invented it based on the term "computer literacy". I used it in the following way: Mathematics literacy is needed, according to the variety of jobs of people who are not specialists in mathematics. Or, we may say: Mathematics literacy is the ground-level knowledge of mathematics users. An official at MEXT at that time perhaps felt uneasy about the term, and he even tried to rephrase it to "application capability". Also, some mathematics scholars who were enthusiastic in education criticized it on the basis that such a term which cannot be translated into Japanese should not be used in the Curriculum Guidelines. Fortunately, Professor Kodaira, after having questioned its position/relation with the concept of numeracy, kindly admitted the use of mathematics literacy.

In the 21st century the importance of talent in mathematics is paraphrased as the call for STEM (Science-Technology-Engineering-Mathematics) education. There, understanding by means of mathematical intelligence and achievements with the help of mathematical methods should be pushed forward together. For those directions, I spoke on the occasion of the Kodaira Prize ceremony, but I would like to reiterate it here: The keyword is multi-layered understanding of concepts both in improving mathematics education of the new era and in developing new fields at the frontier of mathematics. I would define multi-layered understanding as extending from elementary intuition through the deepest grasp of a matter. In conclusion, I would like to direct the reader's attention to the words of Kōan Ogata, a medical doctor who flourished just before the Meiji Restoration and who educated Yukichi Fukuzawa and other talented people in the modern era, that learning must be accomplished with devotion to the people and the country. This attitude is surely beyond the reach of AI, which derives conclusions inductively from big data.

東京

TOKYO City Guide

Irresistible cuisine, Time to shop!, Explore arts and culture,
Tokyo at night, Welcome, sports fans!, Gateway to further adventure

City overview

Tokyo, Japan's bustling capital city, is a modern, vibrant megalopolis that combines business, knowledge, creativity and innovation. The city is the epitome of fusion, where over 400 years of Japanese history is juxtaposed with modern Tokyo, providing a unique experience for all visitors. There is something for everyone: Visitors can choose from over 100,000 restaurants, including more than 200 Michelin-starred restaurants, enjoy one of its 80-plus parks, immerse themselves in the aesthetics of the Japanese tea ceremony or indulge in a night of unique Japanese culture at a Kabuki theatre.

Go ▶ Tokyo website (Food & Drink):
https://www.gotokyo.org/en/see-and-do/drinking-and-dining/index.html

Tokyo Station

Sushi

Irresistible cuisine

The Japanese quest for excellence extends to its kitchens, whether in five-star hotels or the local noodle shops.

Nijubashi Bridge

Food lovers will be pleased to know that washoku, or authentic Japanese cuisine, which includes the globally popular sushi and tempura, has been designated as an Intangible Cultural Heritage in 2013 by UNESCO. There is no better place in the world than Tokyo to enjoy these authentic Japanese delicacies.

Go▶Tokyo website (Shopping):
https://www.gotokyo.org/en/see-and-do/shopping/index.html

Japanese Tea Ceremony

Time to shop!

Tokyo offers countless opportunities for visitors to shop for the beautiful and traditional products for which Japan is famous: exquisite green tea and traditional sweets, elegant lacquerware, etched glass and "Japan-original" designed textiles and fashion. The Ginza shopping district is home to world-renowned fashion and jewellery brands and one of the oldest department stores in the world, Mitsukoshi. The Shibuya/Harajuku area, famous for the Scramble Crossing featured in the film Lost in Translation, is the centre of youth culture and fashion trends in Tokyo. Shinjuku is a district that caters to everyone from businesspeople to students to visitors, and everything from UNIQLO to the renowned department store ISETAN fulfils the needs of locals and tourists alike.

Kanda Shrine

Go▶Tokyo website (Art & Design):
https://www.gotokyo.org/en/see-and-do/arts-and-design/index.html
Go▶Tokyo website (History):
https://www.gotokyo.org/en/see-and-do/history/index.html
Go▶Tokyo website (Culture):
https://www.gotokyo.org/en/see-and-do/culture/index.html

Hagoita (Battledore)

Explore arts and culture

Tokyo has dozens of museums spanning nearly every subject area and interest. For the more contemporary minded, there is an impressive number of smaller museums and galleries showcasing home-grown artists. Museum fans with limited time should consider a visit to Ueno, where a variety of first-class museums, including

The National Museum of Western Art

the Tokyo National Museum, the National Museum of Western Art, the Tokyo Metropolitan Art Museum and the National Science Museum, are located close together.

Tokyo also has 27 theatres, including the Kabuki Theatre, the only theatre in the world dedicated to Kabuki. Kabuki is traditional theatre, performed by men in stunning makeup and gorgeous costumes accompanied by live music. Suntory Hall is home for Tokyo's six professional orchestras. This concert hall is highly regarded by musicians and audiences, with Maestro Herbert von Karajan calling it "a jewel box of sound."

Go ▶ Tokyo website (Nightlife):
https://www.gotokyo.org/en/see-and-do/nightlife/index.html

Shinjyuku

Shibuya Scramble Crossing

Tokyo at night

As dusk falls, Tokyo is transformed into a city of bright lights. Many observation decks in towers and tall buildings are open late, providing great vantage points for stunning night views. An evening stroll through the streets of Shinjuku, Ginza or Shibuya – Tokyo's best-known nightlife districts – is a great way to see the city after dusk and enjoy the myriad of neon lights. The Blue Note Tokyo is the largest club in Tokyo for first-class jazz, Latin and soul acts. Billboard Live – where musicians on their world tours converge – offers a wide range of different genres of music from classic to jazz to pop to R&B. Visitors who want to enjoy a more local atmosphere can patronize the izakaya (local pubs) for a Japanese beer and yakitori.

Yakitori

Welcome, sports fans!

Tokyo is a major centre for sports in Japan. Its professional sports teams compete in baseball, soccer and sumo. Die-hard sumo fans will vouch that sumo wrestling – Japan's traditional national sport – is better than theatre. Intense bouts usually last for just a few intense seconds, with a lot of posturing in between. The centre of sumo in Tokyo is the Ryogoku Kokugikan. The Japanese also love watching professional baseball games and soccer. Visitors who are not familiar with the team or players will still find this very enjoyable as local fans always welcome foreign visitors who join the crowds of spectators.

Sumo

Gateway to further adventure

Thanks to its location at the centre of the country, Tokyo serves as an ideal gateway to Japan. Delegates can enjoy the vast nature and rich history around the country. The country's extensive national transportation system including the Shinkansen (bullet train) makes short getaway trips from Tokyo hassle-free.

Shinkansen and Mt. Fuji

Kyoto and Nara: Two ancient capitals

▶ Kyoto: 135 minutes by bullet train from Tokyo
▶ Nara: less than one hour by local train from Kyoto
Undoubtedly one of the most stunning cities in Japan, Kyoto is the perfect addition to your trip. The Golden Pavilion, the Kiyomizu Temple and Ryoanji with its famous rock garden are magnificent temples listed as UNESCO World Heritage sites. Nara, another former capital, is home to Horyuji Temple, the world's oldest surviving wooden structure at 1,400 years of age. Famous for wagashi, or traditional Japanese sweets, and green tea, Kyoto preserves the atmosphere of traditional Japan whilst also being a hip city popular with young people, many of whom attend its numerous universities.

Horyuji Temple

Arashiyama at Kyoto

Hakone: Onsen spot with a view of Mt. Fuji

▶ An hour and a half by train from Tokyo

Hakone, a popular resort area for locals and visitors alike, is known for its hot springs and beautiful scenery. A 90-minute train ride from Shinjuku will take visitors to the town where Japan's iconic national symbol, the magnificent Mt. Fuji, can be admired from Lake Ashinoko.

Owakudani at Hakone

Nikko: World Heritage site

▶ Two hours by bullet train and local train from Tokyo

Nikko is famous for the shrine dedicated to Tokugawa Ieyasu, the first shogun of the Tokugawa Shogunate. The luxurious colours used on the Toshogu Shrine as well as the Rinno-ji Temple reflect Japanese history in the 17th century. Together with forested areas that include the Nikko-suginamiki-kaido (Cedar Avenue of Nikko, the world's longest tree-lined avenue), Nikko has been designated as a UNESCO World Heritage site and an important cultural property.

Toshogu Shrine

Lake Ashinoko at Hakone

Mt. Takao: Award-winning views

▶ **One hour by local train from Tokyo**

Accessible from Tokyo's city centre in just one hour, Mt. Takao has been awarded 3 stars by Michelin Green Guide Japan. Despite its modest height of 599 m, Mt. Takao is great for casual hiking. The mountain is especially popular during the autumn leaf-viewing season from the middle of November.

A view from the top of Mt. Takao

Hachijo Island: The hot spring island of Tokyo

▶ **50 mins by plane from Haneda airport**

Hachijo Island is one of the nearest island resorts from Tokyo, a gourd-shaped island formed by two mountains to the west and east. The island has subtropical vegetation thanks to the Kuroshio Current, and abundant hot springs. You can enjoy both trekking and diving.

Hachijo Island

Kamakura

▶ **An hour by train from Tokyo**

Capital of Japan during the eponymous Kamakura Period (1185 – 1333), Kamakura is about an hour's train ride from central Tokyo. Renowned for its temples, shrines and the Great Buddha, 11 m in height and dating from 1252, Kamakura is a gorgeous city to explore. Summer is a popular time to visit, when the sandy beaches located on the edge of the city are thronged with bathers.

Great Buddha at Kamakura

Tips for sightseeing spots in Tokyo

Tokyo Skytree

Tokyo Skytree (東京スカイツリー) is a broadcasting and observation tower in Sumida, Tokyo. It became the tallest structure in Japan in 2010 and reached its full height of 634 metersin March 2011.

Tokyo Tower

The Tokyo Tower (東京タワー) is a communications and observation tower in the Shiba-koen district of Minato, Tokyo, Japan, built in 1958. At 332.9 meters, it is the second-tallest structure in Japan.

Asakusa

Asakusa (浅草) is a district in Taitō, Tokyo, Japan.Asakusa has many restaurants and places to try traditional Japanese food.

Imperial Palace

The Imperial Palace (皇居) is the main residence of the Emperor of Japan. It is a large park-like area. The 1.15-square-kilometer (0.44 sq mi) palace grounds and gardens are built on the site of the old Edo Castle.

Tokyo Station

Tokyo Station (東京駅), also sometimes referred to as Tokyo Central Station, is a railway station. The newer Eastern extension is not far from the Ginza commercial district.

Kabukiza Theatre

Kabuki-za (歌舞伎座) in Ginza is the principal theater in Tokyo for the traditional kabuki drama form.

Hamarikyu Gardens

Hama-rikyū Gardens (浜離宮恩賜庭園) is a metropolitan garden. A landscaped garden of 250,216 m^2 includes Shioiri-no-ike (Tidal Pond), and the garden is surrounded by a seawater moat filled by Tokyo Bay.

Shinjuku Gyoen National Garden

Shinjuku Gyo-en (新宿御苑) is a large park and garden in Shinjuku and Shibuya, Tokyo, Japan. It was originally a residence of the Naitō family in the Edo period. It is now a national park under the jurisdiction of the Ministry of the Environment.

Useful Site

- Go Tokyo (The offical Tokyo Travel Guide)
- Visit Chiyoda (Chiyoda City Tourism Association)
- Japan Tourism Agency
- Art & Design in Tokyo
- Wi-Fi & Connectivity

Useful Phrases

Hello	こんにちは	Konnichi wa
Nice to meet you	はじめまして	Hajimemashite
Good morning	おはようございます	Ohayō gozaimasu
Good afternoon	こんにちは	Konnichi wa
Good evening	こんばんは	Konban wa
Thank you	ありがとう	Arigatō
You are welcome	どういたしまして	Dō itashimashite
I am sorry	ごめんなさい	Gomen nasai
Excuse me	すみません	Sumimasen
Yes/No	はい / いいえ	Hai/Iie
How much is it?	いくらですか	Ikura desuka
How do I get to (Place)?	(Place) はどういけばいいですか	(Place) wa do ikeba iidesuka
Where is the restroom?	トイレはどこですか	Toire wa dokodesuka

Memo

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